

Sobolev maps to manifolds

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Sobolev spaces $W^{s,p}$ of maps with values into a compact manifold naturally appear in geometry and material sciences. They exhibit qualitatively different properties from scalar Sobolev spaces: in general, there is no density of smooth maps, and standard trace theory fails. We will present some of their basic properties, with focus on the case $0 < s < 1$, where harmonic analysis tools combined with geometric considerations are quite effective. More specifically, we will address questions concerning the existence of topological invariants, of pullback of forms, the density of nice maps, existence of lifting and extensions. Another topic we discuss is the factorization of unimodular maps, which can be seen as a geometric version of paraproducts.