Limiting Sobolev estimates for vector fields and cancelling differential operators

Jean Van Schaftingen

Institut de Recherche en Mathématique et Physique,

Université catholique de Louvain, Belgium

I will present Sobolev-Gagliardo-Nirenberg endpoint estimates for classes of homogeneous vector differential operators. Away of the endpoint cases, the classical Calderón-Zygmund estimates show that the ellipticity is necessary and sufficient to control all the derivatives of the vector field. In the endpoint case, Ornstein has showed that there is no nontrivial estimate on same-order derivatives and the ellipticity is necessary for endpoint Sobolev estimates. Such endpoint estimates were proved first for the deformation operator (Korn-Sobolev inequality by M.J. Strauss) and for the Hodge complex (Bourgain and Brezis). The class of operators for which estimates holds can be characterized by a cancelling condition. The estimates rely on a duality estimate for L^1 vector fields satisfying some conditions on the derivatives, combined with classical algebraic and harmonic analysis techniques. This characterisation unifies classes of known inequalities and extends to the case of Hardy inequalities.